13.04.2010.

NESVOJSTVENI (NEPRAVI) INTEGRALI

- Nesvojstveni integrali I vrste: (ako je jedna od granica ∞)

a) $\int_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty} \int_a^b f(x)dx$

b) $\int_{a}^{+\infty} f(x)dx = \lim_{b\to+\infty} \int_a^b f(x)dx$

c) $\int_{-\infty}^{+\infty} f(x)dx = \lim_{\substack{a\to-\infty \\ b\to+\infty}} \int_a^b f(x)dx$

Nesvojstveni integral konvergira ukoliko odgovarajući limes postoji i konačan je realan broj. U suprotnom kažemo da ~~interval~~ integral divergira.

* ① $I = \int_2^{+\infty} \frac{dx}{x^2\sqrt{x-1}} = \lim_{R\to+\infty} \int_2^R \frac{dx}{x^2\sqrt{x-1}} = \left\| \begin{matrix} x-1=t^2 \\ dx = 2tdt \\ x = t^2+1 \end{matrix} \quad \begin{matrix} x=2 \Rightarrow t^2=1 \Rightarrow t=1 \\ x=R \Rightarrow t^2=R-1 \Rightarrow t=\sqrt{R-1} \end{matrix} \right\| =$

$= \lim_{R\to\infty} \int_1^{\sqrt{R-1}} \frac{2\cancel{t}dt}{(t^2+1)^2 \cancel{t}} = 2 \lim_{R\to+\infty} \int_1^{\sqrt{R-1}} \frac{dt}{(t^2+1)^2}$

$\int \frac{dt}{(t^2+1)^2} = \int \frac{t^2+1-t^2}{(t^2+1)^2}dt = \int \frac{\cancel{t^2+1}}{(t^2+1)^{\cancel{2}}}dt - \int \frac{t^2 dt}{(t^2+1)^2} = \left\| \begin{matrix} u=t & du=dt \\ dv = \frac{tdt}{(t^2+1)^2} & \\ v = -\frac{1}{2(t^2+1)} & \end{matrix} \right\| =$

$= \text{arctg}\, t - \left(-\frac{t}{2(t^2+1)} + \frac{1}{2}\int \frac{dt}{t^2+1} \right) = \frac{1}{2}\text{arctg}\, t + \frac{t}{2\cdot(t^2+1)} + C$

$I = 2\cdot \lim_{R\to\infty} \left[\frac{1}{2}\text{arctg}\, t + \frac{t}{2\cdot(t^2+1)} \right] \Big|_1^{\sqrt{R-1}} =$

$= 2\cdot \lim_{R\to+\infty} \left[\frac{1}{2}\text{arctg}\sqrt{R-1} + \frac{\sqrt{R-1}}{2(R-1+1)} - \left(\frac{1}{2}\text{arctg}\, 1 + \frac{1}{2\cdot 2} \right) \right] =$

$\frac{\pi}{2}$ $\quad 0$

$= 2\left(\frac{\pi}{4} - \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{4}\right) = 2\cdot\left(\frac{2\pi - \pi - 2}{8}\right) = \frac{\pi - 2}{4}$

② Diskutovati konvergenciju integrala u zavisnosti od zadanih parametara.

a) $\int_1^{+\infty} \frac{dx}{x^\alpha}$

b) $\int_{-1}^{+\infty} e^{-\alpha x} dx$

slično se rješava kao i pod a)

c) $\int_0^{+\infty} e^{\alpha x}(5x+2)\,dx$

d) $\int_0^{+\infty} e^{-ax} \cos bx\,dx$

a) $\int_1^{+\infty} \frac{dx}{x^\alpha} = \lim_{R\to\infty} \int_1^R x^{-\alpha} dx = \lim_{R\to+\infty} \frac{x^{-\alpha+1}}{-\alpha+1}\Big|_1^R = \lim_{R\to+\infty}\left(\frac{R^{1-\alpha}}{1-\alpha} - \frac{1}{1-\alpha}\right)$

$(\alpha \neq 1)$

1° $1-\alpha > 0$ tj. $1 > \alpha \Rightarrow \int_1^{+\infty} \frac{dx}{x^\alpha} = +\infty$

2° $1-\alpha < 0$ tj. $1 < \alpha \Rightarrow \int_1^{+\infty} \frac{dx}{x^\alpha} = -\frac{1}{1-\alpha}$

3° $\alpha = 1$ $\int_1^{+\infty} \frac{dx}{x} = \lim_{R\to\infty} \int_1^R \frac{dx}{x} = \lim_{R\to+\infty} \ln x \Big|_1^R = \lim_{R\to\infty} (\underbrace{\ln R}_{\infty} - \ln 1) = +\infty$

Integral konvergira za $\alpha > 1$, a divergira za $\alpha \leq 1$.

c) $I = \int_0^{+\infty} e^{\alpha x}(5x+2)\,dx = \lim_{R\to\infty} \int_0^R e^{\alpha x}(5x+2)\,dx =$ $\left\| \begin{matrix} 5x+2 = u & v = \int e^{\alpha x}dx = \frac{e^{\alpha x}}{\alpha} \\ 5dx = du & \alpha \neq 0 \end{matrix} \right\|$

$= \lim_{R\to+\infty}\left[(5x+2)\cdot\frac{e^{\alpha x}}{\alpha}\Big|_0^R - \int_0^R \frac{e^{\alpha x}}{\alpha}\cdot 5dx\right] =$

$= \lim_{R\to+\infty}\left[(5R+2)\frac{e^{\alpha R}}{\alpha} - 2\cdot\frac{e^{2\alpha}}{\alpha} - \frac{5}{\alpha}\cdot\frac{e^{\alpha x}}{\alpha}\Big|_0^R\right]$

$$= \lim_{R\to\infty}\left[\frac{e^{\alpha R}}{\alpha}\left(5R+2-\frac{5}{\alpha}\right)-2\cdot\frac{e^{2\alpha}}{\alpha}+\frac{5}{\alpha^2}\right.$$

1° Ako je $\alpha>0 \Rightarrow I=+\infty$

2° Ako je $\alpha<0 \Rightarrow I=\frac{5}{\alpha^2}-2\cdot\frac{e^{2\alpha}}{\alpha}$

$$\lim_{R\to+\infty}\frac{5R+2-\frac{5}{\alpha}}{\alpha R^{-\alpha R}} \overset{L.P.}{=} \lim_{R\to+\infty}\frac{5}{\alpha e^{-\alpha R}\cdot(-\alpha)} = \lim_{R\to+\infty}\frac{5e^{\alpha R}}{-\alpha^2}=0$$

3° Ako je $\alpha=0$:

$$I=\int_0^{+\infty}R^0(5x+2)\,dx=\lim_{R\to+\infty}\left(\int_0^R 5x\,dx+2\int_0^R dx\right)=\lim_{R\to+\infty}\left(\frac{5x^2}{2}\Big|_0^R+2x\Big|_0^R\right)=$$

$$=\lim_{R\to0}\left(\frac{5R^2}{2}+2R\right)=+\infty.$$

Zaključak: Za $\alpha<0$ interval konvergira, a za $\alpha\geq0$ interval divergira

③ $$\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}=\lim_{\substack{a\to-\infty\\ b\to+\infty}}\int_a^b\frac{dx}{1+x^2}=\lim_{\substack{a\to-\infty\\ b\to+\infty}}\operatorname{arctg}x\Big|_a^b=\lim_{\substack{a\to-\infty\\ b\to+\infty}}(\operatorname{arctg}b-\operatorname{arctg}a)$$

$$=\lim_{b\to+\infty}\operatorname{arctg}b-\lim_{a\to-\infty}\operatorname{arctg}a=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)=\pi$$

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④ Dokazati matematičkom indukcijom da vrijedi:

$$\int^{+\infty} x^{2n+1}\cdot e^{-x^2}\,dx=\frac{n!}{2}\quad(n\in\mathbb{N})$$

Tvrdnja vrijedi za $n=1$!

$$\int_0^{+\infty} x^3 e^{-x^2} dx = \frac{1}{2}$$

$x^2 = -t$

$$\int_0^{+\infty} x^3 e^{-x^2} dx = \lim_{R\to+\infty} \int_0^R x^3 e^{-x^2} dx = \left| \begin{matrix} -x^2 = t & x=0 \Rightarrow t=0 \\ -2x\,dx = dt & x=R \Rightarrow t=-R^2 \\ x\,dx = -\frac{1}{2}dt & \end{matrix} \right| =$$

$$= \lim_{R\to+\infty} \int_0^{-R^2} (-t e^t) \cdot \left(-\frac{1}{2}\right) dt \quad \left| \begin{matrix} \text{napomena:} \\ \int t e^t dt = t e^t - e^t + c \end{matrix} \right| = +\frac{1}{2} \lim_{R\to\infty} (t e^t - e^t) \Big|_0^{-R^2} =$$

$$= +\frac{1}{2} \lim_{R\to+\infty} \left((-R^2 e^{-R^2} - e^{-R^2}) - (0 - e^0) \right) = +\frac{1}{2} \lim_{R\to+\infty} \left(\frac{-R^2}{e^{R^2}} + 1 \right)$$

0

$$\overset{L.P.}{=} \frac{1}{2} \left(\lim_{R\to+\infty} \frac{-2R}{e^{R^2} \cdot 2R} + 1 \right) = \frac{1}{2}$$

Pretpostavka:

$$\int_0^{+\infty} x^{2k+1} \cdot e^{-x^2} dx = \frac{k!}{2}$$

važi za neki prirodan broj $k > 1$

Tvrdnja: $$\int_0^{+\infty} x^{2(k+1)+1} e^{-x^2} dx = \frac{(k+1)!}{2}$$

$$\int_0^{+\infty} x^{2k+3} e^{-x^2} dx = \int_0^{+\infty} x^{2k+2} \cdot x \cdot e^{-x^2} dx = \lim_{R\to+\infty} \int_0^R x^{2k+2} \cdot x \cdot e^{-x^2} dx =$$

$$= \left\| \begin{matrix} u = x^{2k+2} \\ du = (2k+2) x^{2k+1} dx \end{matrix} \; ; \; v = \int x e^{-x^2} dx = \left| -x^2 = t \right| = -\frac{1}{2} e^{-x^2} \right\|$$

$$= \lim_{R\to+\infty} \left(-\frac{1}{2} x^{2k+2} e^{-x^2} \Big|_0^R + \frac{1}{2} \int_0^R e^{-x^2} (2k+2) x^{2k+1} dx \right) =$$

$$= \lim_{R\to+\infty} \left(-\frac{1}{2} R^{2k+2} e^{-R^2} + \frac{1}{2} \cdot (2k+2) \underbrace{\int_0^{+\infty} x^{2k+1} e^{-x^2} dx}_{\text{prema (R.) (pretpostavka)}} \right)$$

$$\lim_{R\to+\infty} \frac{R^{2k+2}}{e^{R^2}} \stackrel{L.P.}{=} \lim_{R\to+\infty} \frac{(2k+2)R^{2k+1}}{e^{R^2}\, 2R} \stackrel{L.P.}{=} \dots = \frac{(2k+2)2k(2k-2)\dots 2}{\infty} = 0$$

$$\int_0^{+\infty} x^{2k+3} e^{-x^2} dx = \frac{1}{2}\cdot 2(k+1)\cdot \frac{k!}{2} = \frac{1}{2}(k+1)! = \frac{(k+1)!}{2}$$

Za vježbu:

$\overset{*}{5.}$ $\displaystyle\int_1^{\infty} \frac{dx}{x\sqrt{x^2+x+1}}$ (smjena: $x=\frac{1}{t}$

6. $\displaystyle\int_0^{+\infty} \frac{dx}{x^2+2x+5}$

7. $\displaystyle\int_a^{+\infty} \frac{dx}{x \ln^2 x}$ $(a>1)$

Nesvojstveni integrali II vrste

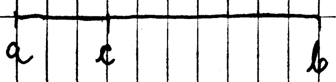
1° $\displaystyle\int_a^b f(x)dx = \left| \begin{array}{l} x=a \text{ je tačka prekida funkcije } f(x) \\ \text{tj. } f(x) \text{ je neograničena u toj tački} \end{array} \right|$

$$= \lim_{\varepsilon\to 0_+} \int_{a+\varepsilon}^b f(x)dx$$

2° $\displaystyle\int_a^b f(x)dx = \left| \begin{array}{c} x=b \text{ je tačka prekida} \\ \text{za funkciju } f(x) \end{array} \right| = \lim_{\varepsilon\to 0_-} \int_a^{b+\varepsilon} f(x)dx =$

$$= \lim_{\varepsilon\to 0_+} \int_a^{b-\varepsilon} f(x)dx$$

3°



Tačka $x=c$ je tačka prekida funkcije $f(x)$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \lim_{\varepsilon\to 0+}\int_a^{c-\varepsilon} f(x)\,dx + \lim_{\delta\to 0+}\int_{c+\delta}^{b} f(x)\,dx$$

(1) $I=\int_0^e x\cdot|\ln x|\,dx = -\int_0^1 x\ln x\,dx + \int_1^e x\ln x\,dx$

$$\int x\ln x\,dx = \left|\begin{matrix} u=\ln x & dv = x\,dx \\ du=\frac{1}{x}dx & v=\frac{x^2}{2}\end{matrix}\right| = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx =$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + C = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$\int_1^e x\ln x\,dx$ - on nije nesvojstven, ne treba mu limes

$$\int_1^e x\ln x\,dx = \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)\Big|_1^e = \frac{e^2}{2}\underbrace{\ln e}_{1} - \frac{e^2}{4} - \left(\frac{1}{2}\underbrace{\ln 1}_{0} - \frac{1}{4}\right)$$

$$= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2+1}{4}$$

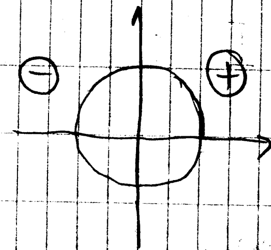
$$\int_0^1 x\ln x\,dx = \lim_{\varepsilon\to 0+}\int_\varepsilon^1 x\ln x\,dx = \lim_{\varepsilon\to 0}\left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)\Big|_\varepsilon^1$$

$$= \lim_{\varepsilon\to 0}\left[\frac{1}{2}\underbrace{\ln 1}_{0} - \frac{1}{4} - \left(\frac{\varepsilon^2}{2}\ln\varepsilon - \underbrace{\frac{\varepsilon^2}{4}}_{\to 0}\right)\right] =$$

$$= \lim_{\varepsilon\to 0+}\left(-\frac{1}{4} - \frac{\varepsilon^2}{2}\ln\varepsilon\right) = \lim_{\varepsilon\to 0}\left(-\frac{1}{4} - \frac{\ln\varepsilon}{2\varepsilon^{-2}}\right) \overset{L.P.}{=}$$

$$= \lim_{\varepsilon\to 0+}\left(-\frac{1}{4} - \frac{\frac{1}{\varepsilon}}{-4\varepsilon^{-3}}\right) = \lim_{\varepsilon\to 0+}\left(-\frac{1}{4} + \underbrace{\frac{\frac{1}{\varepsilon}\varepsilon^{3}}{4}}_{0}\right) = -\frac{1}{4}$$

Zaključak: $I = \frac{1}{4} + \frac{e^2+1}{4} = \frac{e^2+2}{4}$



* (2) $I = \int_0^{\pi} \frac{|\cos x|}{\sqrt{\sin x}}\,dx =$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}\,dx - \int_{\frac{\pi}{2}}^{\pi} \frac{\cos x}{\sqrt{\sin x}}\,dx = \underbrace{\lim_{\varepsilon\to 0+}\int_{\varepsilon}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}}_{I_1} - \underbrace{\lim_{\delta\to 0+}\int_{\frac{\pi}{2}}^{\pi-\delta} \frac{\cos x}{\sqrt{\sin x}}\,dx}_{I_2}$$

$$I_1 = \left| \begin{matrix} \sin x = t^2 & x=\varepsilon \Rightarrow t=\sqrt{\sin\varepsilon} \\ \cos x\,dx = 2t\,dt & x=\frac{\pi}{2} \Rightarrow t=1 \end{matrix} \right| = \lim_{\varepsilon\to 0+}\int_{\sqrt{\sin\varepsilon}}^{1} \frac{2\cancel{t}\,dt}{\cancel{t}} =$$

$$= \lim_{\varepsilon\to 0+} 2t \Big|_{\sqrt{\sin\varepsilon}}^{1} = 2\lim_{\varepsilon\to 0}\big(1-\underset{\substack{\downarrow\\0}}{\sqrt{\sin\varepsilon}}\big) = 2$$

$$I_2 = \lim_{\delta\to 0+}\int_{\frac{\pi}{2}}^{\pi-\delta} \frac{\cos x}{\sqrt{\sin x}}\,dx = \left| \begin{matrix} \sin x = t^2 & x=\frac{\pi}{2} \Rightarrow t=1 \\ \cos x\,dx = 2t\,dt & x=\pi-\delta \Rightarrow t=\sqrt{\sin(\pi-\delta)},\ t=\sqrt{\sin\delta} \end{matrix} \right|$$

$$= \lim_{\delta\to 0+}\int_{1}^{\sqrt{\sin\delta}} \frac{2\cancel{t}\,dt}{\cancel{t}} = 2\lim_{\delta\to 0+} t\Big|_{1}^{\sqrt{\sin\delta}} = 2\lim_{\delta\to 0+}\left(\sqrt{\sin\delta}-1\right) = -2$$

$$I = I_1 - I_2 = 2-(-2) = 4$$

(3) $\int_0^1 \frac{dx}{x\ln^2 x} = \lim_{\substack{\varepsilon\to 0+\\ \delta\to 0+}} \int_{\varepsilon}^{1-\delta} \frac{dx}{x\ln^2 x} = \left| \begin{matrix} \ln x = t & x=\varepsilon \Rightarrow t=\ln\varepsilon \\ \frac{1}{x}dx = dt & x=1-\delta \Rightarrow t=\ln(1-\delta) \end{matrix} \right|$

$$= \lim_{\substack{\varepsilon\to 0+\\ \delta\to 0+}} \int_{\ln\varepsilon}^{\ln(1-\delta)} \frac{dt}{t^2} = \lim_{\substack{\varepsilon\to 0+\\ \delta\to 0+}} \left(-\frac{1}{t}\right)\Big|_{\ln\varepsilon}^{\ln(1-\delta)} =$$

$$= -\left(\lim_{\delta \to 0+} \frac{1}{\ln(1-\delta)} - \lim_{\varepsilon \to 0+} \frac{1}{\ln \varepsilon}\right) = -\left(\frac{1}{0+} - \frac{1}{-\infty}\right) = -(+\infty + 0) = -\infty$$

Zaključak: Ovaj integral divergira.

4. Odrediti α tako da konvergiraju integrali:

a) $I_1(\alpha) = \int_0^1 \frac{dx}{x^\alpha}$

b) $I_2(\alpha) = \int_a^b \frac{dx}{(b-x)^\alpha}$

b) $$I_2(\alpha) = \lim_{\varepsilon \to 0+} \int_a^{b-\varepsilon} \frac{dx}{(b-x)^\alpha} = \left| \begin{array}{lll} b-x = t & dx = -dt & x = a \Rightarrow t = b-a \\ -dx = dt & & x = b-\varepsilon \Rightarrow t = \varepsilon \end{array} \right|$$

$$= \lim_{\varepsilon \to 0+} \int_{b-a}^{\varepsilon} \frac{-dt}{t^\alpha} = \lim_{\varepsilon \to 0+} \int_{\varepsilon}^{b-a} t^{-\alpha}\, dt = \lim_{\varepsilon \to 0+} \frac{t^{1-\alpha}}{1-\alpha}\Bigg|_{\varepsilon}^{b-a} =$$

$$= \lim_{\varepsilon \to 0+} \frac{(b-a)^{1-\alpha} - \varepsilon^{1-\alpha}}{1-\alpha}$$

1° $1-\alpha > 0$ tj. $\alpha < 1$ tada $I_2(\alpha) = \frac{(b-a)^{1-\alpha}}{1-\alpha}$. $\left(\text{jer } \lim_{\varepsilon \to 0+} \varepsilon^{1-\alpha} = 0\right)$

2° $1-\alpha < 0$ tj. $\alpha > 1$ tada $\lim_{\varepsilon \to 0+} \varepsilon^{1-\alpha} = \lim_{\varepsilon \to 0+} \frac{1}{\varepsilon^{\alpha-1}} = \frac{1}{0+} = +\infty$

$I_2(\alpha) = +\infty$, integral divergira

3° $\alpha = 1$

$$I_2(\lambda) = \lim_{\varepsilon \to 0+} \int_{b-a}^{\varepsilon} -\frac{dt}{t} = \lim_{\varepsilon \to 0+} \ln|t| \Big|_{\varepsilon}^{b-a} =$$

$$= \lim_{\varepsilon \to 0+} \Big(\ln|b-a| - \underbrace{\ln \varepsilon}_{-\infty} \Big) = +\infty$$

Interval konvergira za $\lambda < 1$

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(5) Riješiti jednačinu:

$$\int_{\ln 4}^{x} \frac{dt}{\sqrt{e^t - 4}} = \frac{\pi}{4} \quad \to \text{ ovo je nesvojstveni integral} \qquad x = ?$$

$$e^{\ln 4 + \varepsilon} - 4 = e^{\ln 4} e^{\varepsilon} - 4 = 4e^{\varepsilon} - 4 = 4(e^{\varepsilon} - 1) = z^2$$

$$e^{\ln 4} - 4 = 4 - 4 = 0$$

$$I(x) = \int_{\ln 4}^{x} \frac{dt}{\sqrt{e^t - 4}} = \lim_{\varepsilon \to 0+} \int_{\ln 4 + \varepsilon}^{x} \frac{dt}{\sqrt{e^t - 4}}$$

$$\left| \begin{array}{ll} e^t - 4 = z^2 & t = \ln(z^2 + 4) \\ e^t = z^2 + 4 & dt = \frac{2z}{z^2+4} dz \\ t = \ln 4 + \varepsilon \Rightarrow & z = 2\cdot\sqrt{e^{\varepsilon} - 1} \\ t = x \Rightarrow & z = \sqrt{e^x - 4} \end{array} \right|$$

$$= \lim_{\varepsilon \to 0+} \int_{2\sqrt{e^{\varepsilon}-1}}^{\sqrt{e^x - 4}} \frac{\frac{2z}{z^2+4}}{z} dz = 2 \lim_{\varepsilon \to 0+} \int_{2\sqrt{e^{\varepsilon}-1}}^{\sqrt{e^x-4}} \frac{dz}{z^2+4} =$$

$$= 2 \lim_{\varepsilon \to 0+} \frac{1}{2} \operatorname{arctg} \frac{z}{2} \Bigg|_{2\sqrt{e^{\varepsilon}-1}}^{\sqrt{e^x-4}} = \lim_{\varepsilon \to 0+} \left(\operatorname{arctg} \frac{\sqrt{e^x - 4}}{2} - \operatorname{arctg} \frac{2\sqrt{e^{\varepsilon}-1}}{2} \right) =$$

$$I(x) = \operatorname{arctg} \frac{\sqrt{e^x - 4}}{2}$$

$$I(x) = \frac{\pi}{4} \Rightarrow \operatorname{arctg} \frac{\sqrt{e^x - 4}}{2} = \frac{\pi}{4}$$

$$\operatorname{arctg} 1 = \frac{\pi}{4} \Rightarrow \frac{\sqrt{e^x-4}}{2} = 1 \quad \Big| \cdot 2$$

$$\sqrt{e^x-4} = 2 \quad \Big|^2$$

$$e^x-4=4 \Rightarrow e^x = 8 \Rightarrow \boxed{x = \ln 8}$$

Za vježbu:

a) $\displaystyle\int_0^5 \frac{dx}{x^7}$

b) $\displaystyle\int_{-1}^{1} \frac{dx}{\sqrt[3]{x}}$

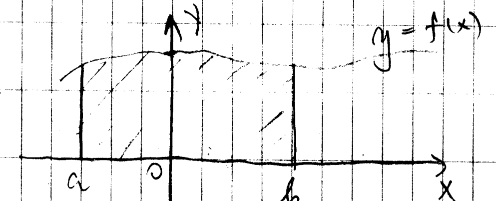
c) $\displaystyle\int_0^1 \frac{dx}{x^3-5x^2}$

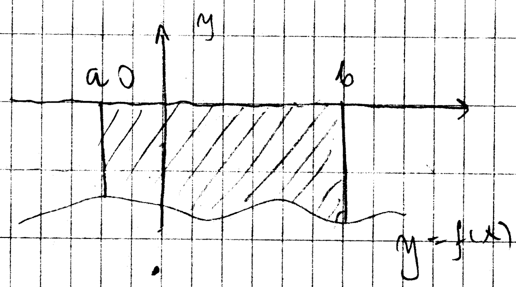
d) $\displaystyle\int_0^2 \frac{dx}{\sqrt{|x^2-1|}}$

e) $\displaystyle\int_0^1 \frac{1}{(2-x)\sqrt{1-x}}$

20. 04. 2010

POVRŠINA RAVNOG LIKA





$y = f(x)$, $x = a$, $x = b$, $y = 0$

$$P = \int_a^b f(x)\,dx$$

$(f(x) \geq 0)$ za $x \in [a, b]$

$f(x) \leq 0$ kad $x \in [a, b]$

$$P = \left| \int_a^b f(x)\,dx \right|$$